MASS TRANSFER IN A PLANE FINITE PORE ON A BROAD INTERVAL OF KNUDSEN NUMBERS WITH ALLOWANCE FOR CONDENSATION

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The flow field and the flow rate of a gas in a two-dimensional plane pore of finite length are found numerically over a broad interval of Knudsen numbers with allowance for condensation on the walls.

Mass transfer processes with allowance for evaporation and condensation on the walls of the system have been investigated in a number of studies, for example, in [1-5]. In [1, 2] the subject of investigation is gas transport between plane infinite plates, in [3, 4] the motion of a gas in an infinite pore is considered within the framework of a series of simplifying assumptions, and in [5] flow in a finite channel is studied but without considering the gas flow rate and its dependence on the channel length and the flow regime. In this article we investigate the mass transfer kinetics in a plane finite pore over a broad interval of Knudsen numbers with allowance for the processes of evaporation and condensation on the pore walls.

Let us consider a plane pore of height 2a and length l infinite in the z direction (see Fig. 1). The pore wall temperature is kept constant. We introduce the following scales: a — length, $\beta^{1/2} = (2RT)^{1/2}$ — velocity, n_e — numerical density, $n_e\beta^{-3/2}$ — distribution function, and $\eta_e = (1/2)n_e m v \lambda_e$ — viscosity coefficient. In what follows all the expressions are written in dimensionless form. It is assumed that all the molecules reaching the surface are condensed, and that those evaporating from the walls have a Maxwellian distribution function with saturated vapor density at the wall temperature $f_e = \pi^{-3/2}\exp(-c^2)$. The pore is connected with a vessel of infinite volume in which the constant density n_a is maintained and $|n_a - 1| = |\Delta n| \ll 1$. The molecules entering the pore through its mouth have a distribution function $f_a = n_a f_e$. In reality, on the preentrance interval the distribution function varies, but taking this variation into account is a complex problem which goes beyond the bounds of this study. It is required to find the flow field in the pore and the rate of flow of gas through its mouth.

As the initial equation for the distribution function we will take the BGK model of the Boltzmann equation [6], which with our scales has the form:

$$\mathbf{c}\frac{\partial f}{\partial \mathbf{r}} = \delta(f^0 - f), \tag{1}$$

where

$$f^0 = n(\mathbf{r})\exp[-(\mathbf{c} - \mathbf{u}(\mathbf{r}))^2];$$
$$n(\mathbf{r}) = \int f(\mathbf{r}, \mathbf{c})\,d\mathbf{c}; \quad \mathbf{u}(\mathbf{r}) = \int f(\mathbf{r}, \mathbf{c})\,\mathbf{c}\,d\mathbf{c}. \tag{2}$$

In accordance with our assumptions, the boundary conditions are

$$\begin{aligned} &x = 0, \ -1 < y < 1, \ c_x > 0, \ f = f_a; \\ &x = L, \ -1 < y < 1, \ c_x < 0, \ f = f_e; \\ &0 < x < L, \ y = \pm 1, \ c_y \lessgtr 0, \ f = f_e. \end{aligned} \tag{3}$$

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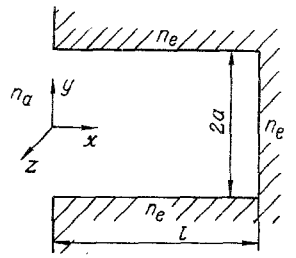

Fig. 1. Geometry of the problem.

As the density difference is small, we can represent the unknown distribution function in the form:

$$f = f_e[1 + h(\mathbf{r},\ \mathbf{c})\Delta n]. \tag{4}$$

Substituting (4) in (1) and linearizing, we easily obtain the expression for the perturbation function

$$h(\mathbf{r},\ \mathbf{c}^*) = \delta \int_0^{s_0} [(n-1) + 2(\mathbf{c}^* \cdot \mathbf{u})] \exp\left(-\frac{\delta s}{c^*}\right) \frac{ds}{c^*} + h_b \exp\left(-\frac{\delta s_0}{c^*}\right), \tag{5}$$

where s_0 is the distance from the observation point to the boundary in the direction $\mathbf{c}^* = \mathbf{c}_x + \mathbf{c}_y$; and h_b is the perturbation function of the molecules traveling from the boundary towards the observation point. Substituting (4) in (2) with allowance for (5) and introducing the notation

$$\begin{aligned} q_1(\mathbf{r}) &= (n(\mathbf{r}) - 1)/\Delta n = \int f_e h d\mathbf{c}, \\ q_2(\mathbf{r}) &= u_x(\mathbf{r})/\Delta n = \int f_e h c_x d\mathbf{c}, \\ q_3(\mathbf{r}) &= u_y(\mathbf{r})/\Delta n = \int f_e h c_y d\mathbf{c} \end{aligned} \tag{6}$$

for the moments of the distribution function, we can write out a system of two-dimensional Fredholm integral equations of the second kind

$$q_i(\mathbf{r}) = \int_{-1}^{1} \int_0^L \sum_{y=1}^{3} K_{ij}(\mathbf{r},\ \mathbf{r}')\, q_j(\mathbf{r}')\, d\mathbf{r}' + Q_i(\mathbf{r}), \quad i = 1,\ 2,\ 3, \tag{7}$$

where

$$K_{11} = \frac{\delta}{\pi s} I_0(\delta s);\ K_{22} = \frac{2\delta}{\pi s^3}(x - x')^2 I_2(\delta s);$$

$$K_{12} = \frac{2\delta}{\pi s^2}(x - x')\, I_1(\delta s);\ K_{23} = \frac{2\delta}{\pi s^3}(x - x')(y - y')\, I_2(\delta s);$$

$$K_{13} = \frac{2\delta}{\pi s^2}(y - y')\, I_1(\delta s);\ K_{33} = \frac{2\delta}{\pi s^3}(y - y')^2 I_2(\delta s);$$

$$K_{21} = \frac{1}{2} K_{12};\ K_{31} = \frac{1}{2} K_{13};\ K_{32} = K_{23};$$

$$s = [(x - x')^2 + (y - y')^2]^{1/2};\ I_n(\delta s) = \int_0^{\infty} c^n \exp\left(-c^2 - \frac{\delta s}{c}\right) dc;$$

$$Q_1 = \frac{1}{\pi} \int_{-\alpha_1}^{\alpha_2} I_1\left(\frac{\delta x}{\cos\varphi}\right) d\varphi;\ Q_2 = \frac{1}{\pi} \int_{-\alpha_1}^{\alpha_2} I_2\left(\frac{\delta x}{\cos\varphi}\right) \cos\varphi\, d\varphi;$$

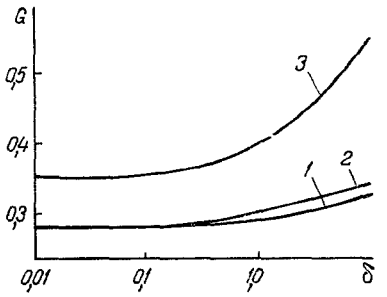

Fig. 2. Rate of flow of gas through pore mouth G (dimensionless quantity) as a function of the reciprocal Knudsen number δ (dimensionless quantity): 1) L = 2; 2) L = 10; 3) data of [4].

$$Q_3 = \frac{1}{\pi}\int_{-\alpha_1}^{\alpha_2} I_2\left(\frac{\delta x}{\cos\varphi}\right)\sin\varphi d\varphi;$$

$$\alpha_1 = \operatorname{arctg}\frac{1-y}{x};\ \alpha_2 = \operatorname{arctg}\frac{1+y}{x}.$$

System (7) was solved numerically by the Krylov–Bogolyubov method [7]. The calculations were carried out on a EC-1052 computer with an accuracy no worse than 2%. The flow field was found on the interval $0 < \delta \leq 10$ for L = 2, 10, 20.

Considerable practical importance attaches to the rate of flow of gas through the mouth of the pore

$$G = \frac{1}{2\Delta n}\int_{-1}^{1} u_x|_{x=0}dy. \qquad (8)$$

From the definition of the gas velocity (2) and the boundary conditions (3) it is easy to show that in the free-molecular regime the flow rate G does not depend on the length L. Within the stated limits of accuracy this assertion can be extended to fairly large but finite Knudsen numbers. On the other hand, for any Knudsen number, as the ratio of the pore length to the mean free path increases the influence of the "bottom" on the flow near the mouth and hence on the gas flow rate decreases. Thus, over the entire range of Knudsen numbers there is a pore length such that with given accuracy a further increase in pore length does not lead to a change in flow rate. An analysis of the numerical calculations shows that the difference in flow rates for L = 10 and L = 20 lies within the margin of error of the calculations for all Knudsen numbers, i.e., the results obtained for L = 10 can be extended to any greater length, up to infinity, with an error not worse than 2%.

In Fig. 2 we have plotted the flow rate G against the reciprocal Knudsen number δ for various pore lengths and have compared our results with the data of [4], where an infinite pore with phase transitions at the walls was investigated, but an unjustified simplifying separation of variables in physical space was introduced for the perturbation function h, and the system of equations was solved by the Bubnov–Galerkin variational method. Clearly, the flow rate versus reciprocal Knudsen number curves are in only qualitative agreement. The difference is 40% in the viscous regime and 20% in the free-molecular regime. Analyzing the numerical results obtained for the flow field we note that in any regime condensation is most intense on the initial section of the pore; half the gas entering through the pore mouth is condensed over a distance of one pore width from the end face, and 75% of the gas over a distance of two pore widths from the end face.

These conclusions can be used for predicting the characteristics of technical apparatus.

NOTATION

a, half-width of the pore; l, pore length; $L = l/a$, reduced pore length; $\mathbf{r} = r(x, y)$; m, molecular mass; $v = (8RT/\pi)^{1/2}$, thermal velocity of the molecules; T, pore wall temperature; λ, mean free path; R, gas constant; n, dimensionless gas density; Δn, dimensionless density

difference; f, dimensionless distribution function; f^0, local Maxwellian distribution function; **c** and **u**, reduced velocities of the molecules and the gas, respectively; $\delta = \sqrt{\pi}a/2\lambda$, reciprocal Knudsen number; and G, reduced flow rate of gas through the pore mouth. Subscripts: e, saturated vapor; a, gas in the infinite vessel with which the pore is connected; x and y, longitudinal and transverse components.

LITERATURE CITED

1. T. Soga, Phys. Fluids, No. 25 (11) (1982).
2. T. Soga and Y. Osaki, Trans. Jpn. Soc. Aero. Sci., 26, No. 74, 187-197 (1984).
3. N. V. Pavlyukevich, G. E. Gorelik, V. V. Levdanskii, et al., Physical Kinetics and Transport Processes in the Presence of Phase Transitions [in Russian], Minsk (1980).
4. F. M. Sharipov and T. V. Shchepetkina, in: Molecular Physics of Nonequilibrium Systems [in Russian], Novosibirsk (1984), pp. 15-20.
5. P. Gajewski and A. Wisniewski, Bull. Acad. Pol. Sci. Ser. Sci. Tech., 26, No. 5, 513-520 (1978).
6. P. L. Bhatnagar, E. P. Gross, and M. A. Kroock, Phys. Rev., 94, No. 3, 511-520 (1954).
7. L. V. Kantorovich and V. I. Krylov, Approximate Methods of Higher Analysis [in Rusaian], Moscow–Leningrad (1952).

INVESTIGATION OF VAPOR PHASE DYNAMICS IN THE FILM BOILING OF NITROGEN IN A CENTRIFUGAL FORCE FIELD

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The wavelength and the bubble separation diameter and frequency have been measured for the film boiling of liquid nitrogen on a thin horizontal wire at acceleration loads $\eta \leq 375$. A description of the process is given. The results obtained are compared with the available experimental and theoretical data.

The widespread use of cryogenic fluids in the national economy requires a comprehensive study of the question of heat transfer in relation to various regime parameters (pressure, subcooling, acceleration load). In order to calculate the unsteady processes associated with the chilling of cryogenic devices in a centrifugal force field it is important to have information on the film boiling regime and, in particular, the vapor phase dynamics.

A visual study of the film boiling of nitrogen at various pressures, subcoolings and cylindrical heater dimensions was carried out under conditions of normal gravity in [1]. The effect of various parameters (heat flux, heater diameter, acceleration load, etc.) on the thermal and hydrodynamic characteristics of the film boiling process was considered for noncryogenic fluids in [2, 3]. Experimental data on the vapor phase dynamics associated with the film boiling of cryogenic fluids in a centrifugal force field, i.e., under the combined influence of acceleration load η, liquid subcooling Φ, and pressure P, are still lacking.

This investigation of the film boiling of nitrogen was carried out on the apparatus described in [4] (centrifuge with vertical axis of rotation) on the acceleration load interval $1 \leq \eta \leq 375$ and the heat flux interval $q_{cr2} \leq q \leq 4q_{cr1}$. As the heat-transfer surface we used a Nichrome wire $2.2 \cdot 10^{-4}$ m in diameter and $3.8 \cdot 10^{-3}$ m long arranged at right angles to the centrifugal acceleration vector and the axis of rotation. The heat-transfer surface was not specially cleaned after heating at heat fluxes up to 3-4 times greater than q_{cr1}.

At the level of the heat-transfer surface the pressure and subcooling of the liquid nitrogen increased with increase in the acceleration load. In the experiments the greatest subcooling was about 8°K, the greatest pressure $P = 2.5 \cdot 10^5$ N/m^2. We also carried out measurements at constant acceleration loads and various values of the subcooling, which depended on the thickness of the film on the heat-transfer surface.

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